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PAPER TEMPLATE

A CASE STUDY ABOUT AN ACTIVITY OF MEASUREMENT ESTIMATION OF VOLUME.

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In this contribution we present a study about how 27 in-service primary school teachers do a pedagogical value of an activity about measurement estimation of volume for kids (9 years old) proposed in Chileans governmental curricular orientations. In this paper, we analysis developed in our study is supported by a reconstruction of the concept of measurement estimation and a part of Mathematics Teachers' Specialised Knowledge (MTSK). The results show that the knowledge of a significant part of the participant teachers presents confuse measurement with measurement estimation.

INTRODUCCIÓN

There is a broad consensus on measurement estimation as a subject that should be part of the school curriculums (Callís, 2006). However, this topic is not a subject that has received adequate attention from research in mathematics education (Sowder, 1992; Hogan y Brezinski, 2003; Callís et al., 2006). Also the few existing studies about measurement estimation focus on longitudinal magnitudes. In Chile, the measurement estimation has been introduced as content of the new primary curriculum, The Curricular Bases 2012. For these reasons we want to investigate the knowledge about pedagogical possibilities that teachers attribute to an activity about measurement estimation of volume proposed by the Chilean Ministry of Education's guidelines.

FRAMEWORK

In our studies we use the following definition of measurement estimation: "To perceptively assign a value, or an interval of values, together with a unit that corresponds to a discrete or continuous magnitude, by means of previous knowledge or by an indirect comparison to an auxiliary object" (Pizarro, Gorgorió & Albarracín, 2014, p. 528). This definition is based on three essential elements: assigning a numerical value (V), executing the task perceptively (P) and relating perception to previous knowledge or to the mental image of an auxiliary object as a referent (R). These elements should be considered when we assess a mathematical activity like estimation of measurement.

About teacher's knowledge framework, we use the Mathematics Teachers' Specialised Knowledge (MTSK) proposed by Carillo, Flores-Medrano, Escudero-Ávila, Aguilar & Montes (2014). We consider two of the different subdomains of MTSK, specifically we focus on Knowledge of Topic (KoT) and Knowledge of Mathematics Teaching (KMT).

KoT is characterized by focusing on knowledge of the mathematical content and its meaning. KoT contains both, the expected knowledge of the students and a deep knowledge of the mathematical topics. For its part, KMT focuses on the knowledge of materials, methods for presenting contents and its potential impact for instruction and the knowledge of appropriate examples for teaching each content.

In Mathematics Education literature, there are only a few studies focused on teaching of measurement estimation. Forrester & Piké (1998) appreciate a notorious gap between measurement and measurement estimation in the classroom discourse of teachers. In this way, measurement estimation is taught as predictive hypothesis and as a vague and superfluous task. Possibly, this situation is related to the lack of research on the topic, resulting in a weakness in teaching (Joram et al., 1998), because there are no precise guidelines for the teaching of the measurement estimation (Frías, Gil & Moreno, 2001).

Jones, Forrester, Gardner, Grant, Taylor & Andre (2012) inquired about how the students estimates. In their conclusions, they noted that it is not known if teachers are really teaching skills to estimate implicitly or explicitly, nor whether this affects the development of measurement estimation skills by the students. Hogan & Brezinski (2003) affirm that research on measurement estimation consider mostly to the longitudinal measurement estimation. In this way, our research objective is to characterize the pedagogical value performed by teachers on an activity about measurement estimation of the volume.

THE STUDY

Our research is a case study. We interview 27 in-service primary school teachers from Chile. The Chilean Ministry of Education provides plans and programs of study for all school leves. These guideline are for teachers can interpret and observe examples of the prevailing curriculum. We take only activity about measurement estimation of volume in programs of study for primary. This activity is for fourth level with kids of 9 years-old. Then, we ask to teachers: Do you consider this activity appropriate to teach the measurement estimation of volume? Referring to Fig. 1 activity.

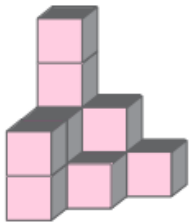
	<p>Suggested assessment indicators: <i>Students estimate and check the volume of irregular objects, plunging them into a graduated beaker.</i></p> <p>Activity: <i>Measure and write down the volume of the figure.</i></p> <p>Evaluation criteria: <i>students identify a unit for measuring volume. Students count all cubes of figure</i></p>
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Fig 1. Activity of measurement estimation of volume

We consider interesting to note that there are a contradiction between activity and assessment indicators, because the first one suggests to estimate and compare and the second one request measure and write down. On the other hand, the object used is not really an irregular object, because if we think of the image in three dimensions we realize that the object is composed by 12 cubic units. In the same way, the beaker proposed is a measuring instrument; therefore, a measurement is performed. In addition, it's necessary to count cubes and , as a mathematical task, it's a way to make a measurement of a discrete magnitude

DATA ANALYSIS

Considering the definition of measurement estimation and the comments made in the previous section, we analyze the responses of 27 teachers. From this analysis emerge four categories as detailed below:

A. Teachers would use the activity like a count of cubes

Five of the 27 teachers use the activity, because they understand the measurement estimation like to count. They explain that they would do activity using counting and a difficulty for students is the spatial orientation. For instance, one of the teachers answers:

María: *"I would use this activity without problems, but students would have problems with spatial orientation, because there are eight visible cubes, there is one cube that is behind... I do not know, those things are complicated for students"*.

B. Teachers say that this activity is about count or measurement; it is not an activity about estimate measurement.

Fifteen teachers are part of category. Some teachers (six) explain that this activity is about count cubes, as in the following examples of response, another teachers say that this activity is a measurement, because it involves a measuring instrument, as shown in the following examples of response:

Alicia: *"This activity is about count. It is not necessary to estimate measurement"*.

Manuel: *"No, because the object is measured in a graduated beaker. It is not an estimation, this is a precise measurement"*

In this way, Alicia thinks that when a count of the buckets exists, measurement estimation does not take place and Manuel thinks that in an activity of estimation of measure must not there be an instrument of measure.

C. Teachers change the activity

There are eight teachers who decide to change the proposed activity (including one teacher of group B). They create another activity that can contain learning opportunities. However, these new tasks designed are just measurement activities, as shown in the following examples of response:

Alejandro: *"I would use a shoebox and smaller cubes. Then, students would must put the cubes into shoebox until complete all its capacity"*

In this response, we note that the teacher creates an activity about measurement of volume by use of right prisms. Another teacher answer is the following:

Andrea: *"I have a graduated beaker, then I would put stones into it, and I say "ok, this measured is x cubic centimeters""*.

This teacher changes the measurement object, however, her proposal to the students is to perform a measurement.

CONCLUSIONS

In this contribution we can see that KoT about measurement estimation presents some contradictions both in the ministerial guidelines and pedagogical values given by teachers. We understand that the lack of perception processes or the use of references in the tasks shows a deficit in the KoT of interviewed teachers.

We can observe in the teachers' answers that measurement estimation of a volume is characterized by develop a task about volume measurement or to count cubes. In this way, the aim of the activity

proposed in the curriculum would not develop the desired mathematical skills, because teachers do not consider neither a reference or perception to guide the classroom work.

In order to solve this lack of knowledge situation, we consider that measurement estimation should be included in teacher-training curricula. It would be interesting for pre-service teachers, and for in-service teachers too, to receive training to include activities that required for them to carry out their own estimations and compare them to measurement processes. It would also be useful for teachers to observe mathematical processes that distinguish the different types of estimation presented in our literature review, as a way of reinforcing their specific mathematical content. If not, this a weakness in teacher's practice could be perpetuated in the Chilean educational system (Joram et al., 1998; Frías, Gil & Moreno, 2001).

From our perspective, teacher changing the activity is a good initiative, because it is a sign of critical spirit. Unfortunately without a solid KoT, these intentions do not come to fruition. Therefore, it is essential to continuing training teachers in measurement estimation.

From this brief analysis on the case of an activity of measurement estimation of volume, we propose to analyze the activities of estimation of different magnitudes into the school curriculum of the diverse educational realities, in order to investigate in how the concept and its skills are developing in the classrooms of our respective countries.

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